Wood End Primary School

Calculations Policy
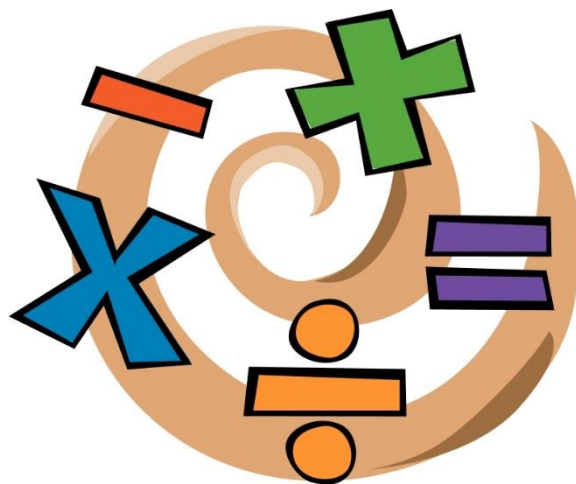
May 2019

RECEPTION CALCULATION GUIDANCE: ADDITION, SUBTRACTION, MULTIPLICATION and DIVISION

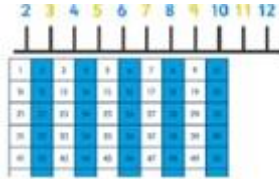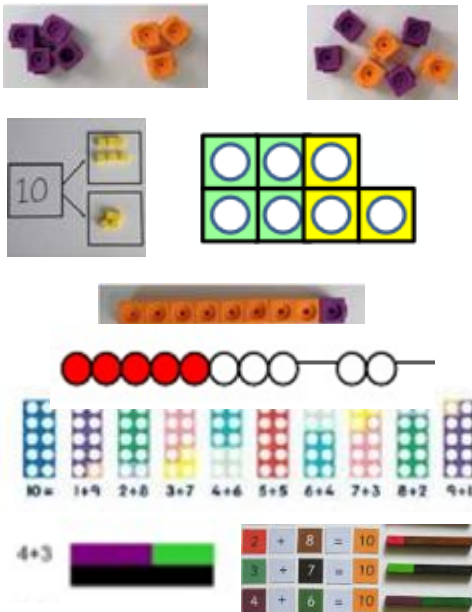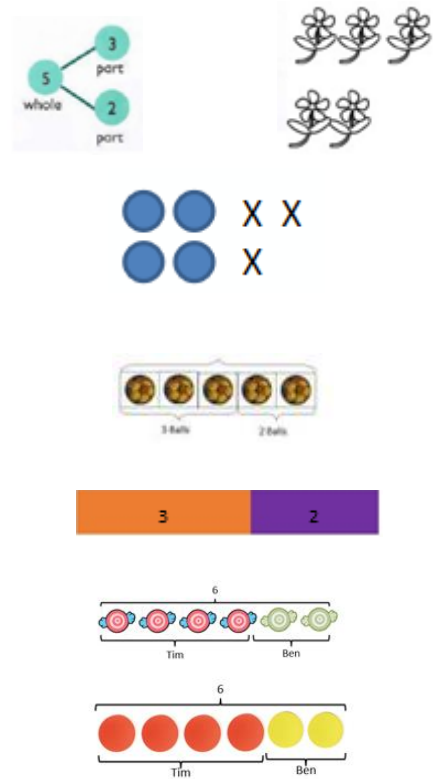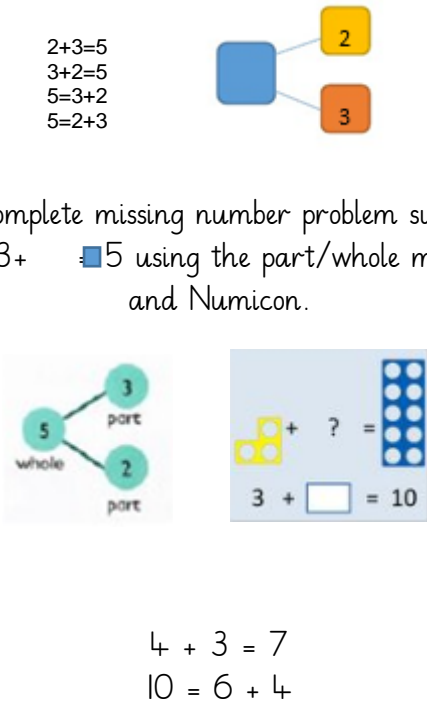| Children are encouraged to develop a mental picture of the number system in their heads to use for calculation				
	Addition	Subtraction	Multiplication	Division
Reception	<u>Progression of Calculation</u> Ones – Ones (combining objects) 1 more than a given number up to 20 Ones + Ones – counting on from a given number Children develop ways of recording calculations using Numicon, bead strings, counters, pictures, marks etc. Children use Number Lines (to 10 and beyond) and practical resources to support calculation and teacher demonstrates the use of the Number Line.	<u>Progression of Calculation</u> Ones – Ones (take away) 1 less than a given number Ones – Ones (comparison E.g. How many more, How many less) Children develop ways of recording calculations using Numicon, pictures, words, fingers, counters etc.	<u>Progression of Calculation</u> Counting in 2s and 10s Beginning to double single digit numbers Children will experience equal groups of objects using counting equipment, Numicon, Cuisenaire etc. Children begin to record doubles	Creating equal groups of a set of objects Sharing a set of objects Children will understand equal groups and share items out in play and problem solving. Explore sharing into equal groups and sets with counting equipment, Numicon, Cuisenaire.

Children experiment with combining different Numicon tiles together to find a total or match another piece.

8+2=10

Children use Number Lines, tracks and Numicon shapes to find one less and to support with counting back. Teachers demonstrate the use of the Number Line.

Children use objects and Numicon to help them compare objects and to say how many more or less.

Children use songs, games and real life contexts to count in repeated groups of the same size. (2s, 10s)

Children use number squares, tracks to begin counting in groups.

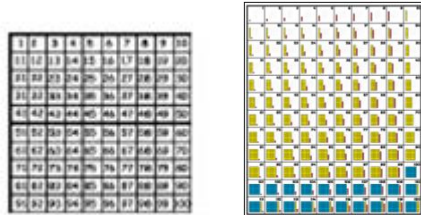| Key language	Add More Total Make Sum	Take Away Less Left Over Fewer Difference Between	Double	Share Halving
Key Resources	Numicon Counting Equipment Bead Strings Number Lines	Bead Strings Number Lines Counting Equipment	Numicon Cuisenaire Number Lines	Numicon Cuisenaire Cubes
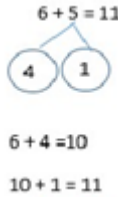
	Objective	Concrete	Pictorial	Abstract
Year 1	Number bonds of 5, 6, 7, 8, 9 and 10 Begin to represent for bonds to 20 Y1 T11	Use cubes or Numicon to add two numbers together as a group or in a whole group or in a bar.	Use pictures to add two numbers together as a group or in a bar.	Use the part-part-whole diagram as shown above to move into the abstract. 2+3=5 3+2=5 5=3+2 5=2+3 Complete missing number problem such as 3+ ☐ =5 using the part/whole model and Numicon. 4 + 3 = 7 10 = 6 + 4

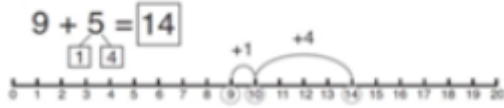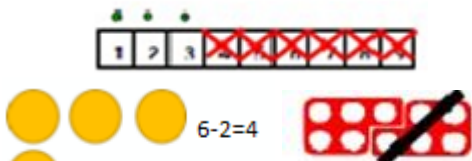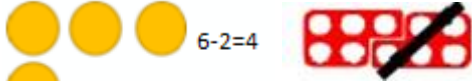|				
		Use Cuisenaire/Numicon tiles to calculate addition 16 + 4 17 + 3 18 + 2 19 + 1		
	Counting to 20 Begin to count in 10s from a single digit number.	Start with the larger number on the bead string/ Numicon and then count on to the smaller number 1 by 1 to find the answer. 5 6 7 8 12 + 5 = 17	Use a number line to count on in ones. 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 +1 +1 +1 +1 +1 6 7 8 9 10 11 12 13 5 6 7 8 Begin to draw their own empty number lines to support calculation. 13 18	Place the larger number in your head and count on the smaller number to find your answer 5 + 3 = 8 8 + 5 = 13 10 + 4 = 15
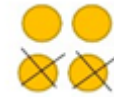
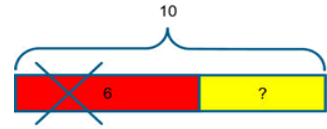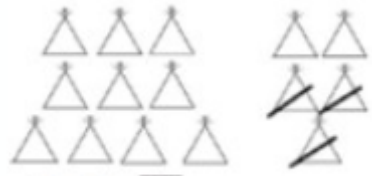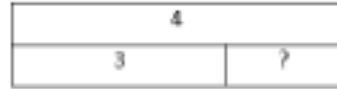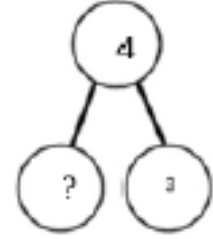|		Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	Become familiar with the 100 square including a Numicon number square — quickly find where a number is located. Know that we move to the right when we count in ones and downwards when adding 10.	
	Regrouping to make 10 (for calculations up to 20) Y1 T12	Start with the bigger number and use the smaller number to make 10 9 + 3 = 12 6 + 5 = 11 	Use pictures or a number line. Regroup or partition the smaller number to make 10 6 + 5 = 11 4 1 6 + 4 = 10 10 + 1 = 11	If I am at 7, how many more do I need to make 10? How many more do I add on now? 7 + 4 = 11

3 + 9 =

9 + 5 = 14

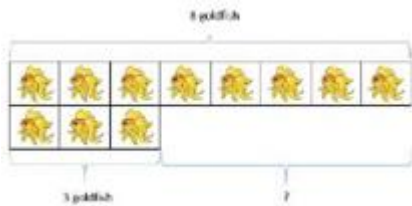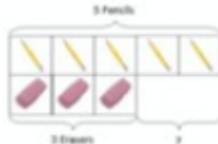| Progression of Calculation	Key Language/ Symbols	Key Maths Resources
O+O counting on O+O crossing 10 O+O crossing 10 using number facts to bridge Teens + O within 20 O + multiple of 10 1 more than any give number to 100 O = ones	Addition Equal to Same as Number bonds Missing Number Sum Total Parts and wholes Plus Add Altogether	Numicon Bead Strings Number lines Counting equipment Cuisenaire Hundred Square Tens Frame Cubes Place Value arrows

YEAR 1 CALCULATION GUIDANCE: SUBTRACTION
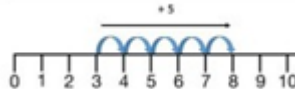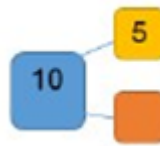
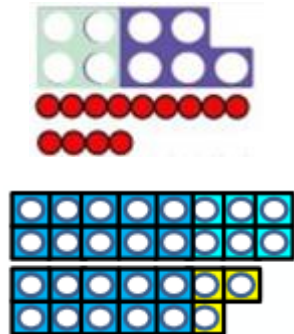|	Objective	Concrete	Pictorial	Abstract
Year 1	Taking away ones (from 20) Y1 T13	Use physical objects, counters, cubes etc. to show how objects can be taken away. $4 - 2 = 2$ $9 - 6 = 3$ 6-2=4 $10 - 5 = 5$	Cross out drawn objects to show what has been taken away. 10 6 ? $15 - 3 = \boxed{12}$	$4 - 2 = 2$ $10 - 6 = 4$ 4- 3 = $\boxed{} = 4 - 3$ 4 3 ?
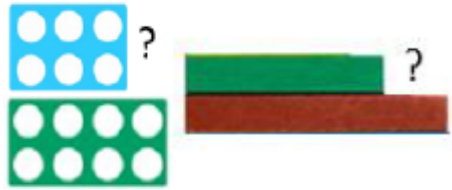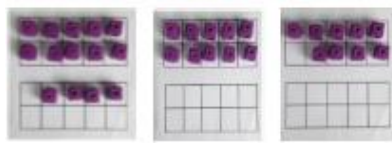
Subtraction using counters   10 - 3 = 7

| Counting back from 20 (including bridging through 10 using number bonds to help)

Begin to count back in 10s from a number between 11 and 19

Y1 T13 | Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.

Use counters and move them away from the group as you take them away counting backwards as you go.

Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting | Count back on a number line or number track

Start at the bigger number and count back the smaller number showing the jumps on a number line.

Become familiar with the 100 square – quickly find where a number is located.
Know that we move upwards when subtracting 10 and left when subtracting 1.

 | $13 - 4 = 9$

Put 13 in your head, count back 4.

What number are you at?

Use your fingers to help.
$13 - 5 = 8$

$19 - 10 = 9$ |

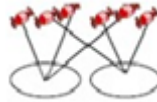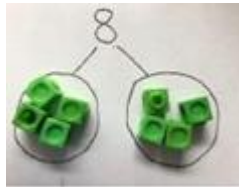|		back 2. $(13 - 5 = 8)$		
	Find the difference (calculations up to 20) Y1 T13	Compare amounts and objects to find the difference Use basic bar models with items to find the difference. Use cubes to make towers Use Numicon to find the difference 9 − 4 and bead string bar models.	Count on to find the difference Use a number line to show that subtraction can also mean finding the difference e.g. $9 - 4 =$ can also mean 'the difference between 4 and 9' and how many jumps they are apart. Draw bars, cubes and other concrete objects they have used to find the difference between 2 numbers.	Hannah has 8 goldfish. Helen has 3 goldfish. Find the difference between the number of goldfish the girls have. $8 - 3 = 5$ $9 - 4 = 5$ $10 - 6 = 4$ Complete missing number problems such as $10 - 5 = ?$ using the part/whole model.

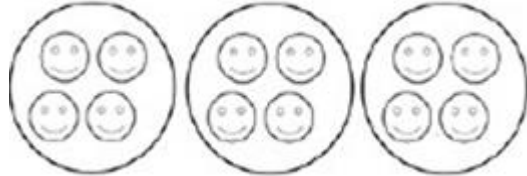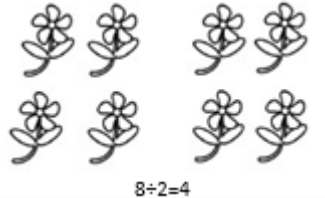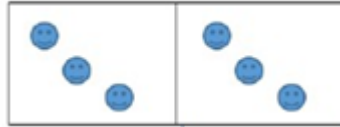|			
	Use Cuisenaire to find the difference.	Tom 10 Sam 6 ? XXXXXXXX XXXXX Use of the bar model ?	
	Making Ten	Using Numicon and Ten Frames 14 – 5 =	Children to present the ten frame pictorially	14 – 5 = 9 You also want children to see related facts e.g. 15 – 9 = 5 Children to represent how they have solved it e.g. 14 – 5 = 9 14 is made up of 5, 5 and 4 so I can subtract one 5 to be left with 4 and 5 5 5 14 – 5 = 9 5 is made up of 4 and 1 so I can subtract 4 to make 10 and then 1 to get to 9 4 1
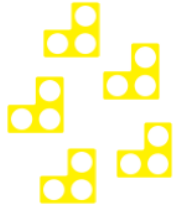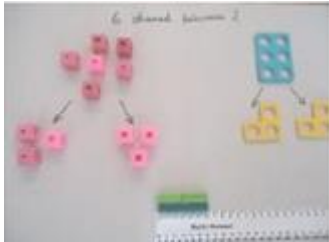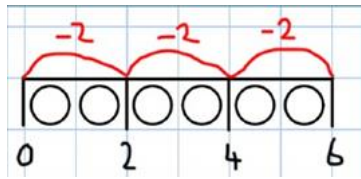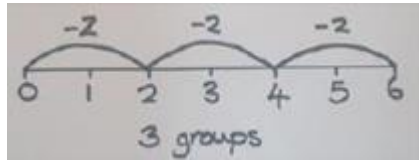
		Children could also do this by subtracting a 5 from the 10		
		Progression of Calculation O-O (take-away) O - O (comparison and difference) Numbers to 20 – O Numbers to 20 – O (Crossing 10) Any subtraction from 20 O = ones	Key Language/ Symbols Subtract Equals Difference Number bonds Missing Number Take away Less than Minus Fewer Decrease	Key Maths Resources Bead Strings Number lines Counting equipment Cuisenaire Hundred Square Cubes Numicon Ten Frames

YEAR 1 AND YEAR 2 CALCULATION GUIDANCE: DIVISION

	Objective	Concrete	Pictorial	Abstract
Year 1 and Year 2	Sharing Y1 T20 Y2 T20	Children use physical objects to share equally. 6 sweets shared between 2 people, how many do they each get? I have 8 cubes, can you share them equally between two people? 	Children use pictures or shapes to share quantities. 8÷2=4 Sharing can also be done in a bar so all 4 operations have a similar structure: 	Share 8 buns between two people. 8÷2= 4 What's the calculation? | 3	3 | 6÷2=3

13

15 divided by 5

	Grouping or Repeated Subtraction Y2 TI6 Y2 TI7 Y2 TI8	Repeated subtraction using a bead bar: $12 \div 3 = 4$ Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding. $12 \div 4 = 3$	Use a number line to show repeated subtraction jumps. The number of jumps equals the number of groups. $10 \div 2 = 5$ 12 children get into teams of four. How	Divide 10 into 5 groups. How many are in each group? $10 \div 5 = 2$ Abstract number line

12

20 ÷ 4 = 5

15 ÷ 5

14÷2=7

many teams will there be?

Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.

10

?

10 ÷ 5 = ?

5 x ? = 10

Progression of Calculation	Key Language/ Symbols	Key Maths Resources
Year 1: Halving 0-20 0-20 ÷	Half Halve Division Dividing	Cubes Counters Arrays Numicon

		Year 2: Halving 0-40 TO÷O(using jottings and equipment in context) TO÷2,5,10 (using known facts)	Grouping Sharing Divided by Divided into Equal groups of Is equal to/ is the same as	Bead Strings Number Lines

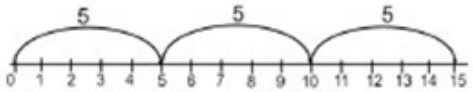|	Objective	Concrete	Pictorial	Abstract
Year 1 and Year 2	Repeated addition: Counting in 2s, 5s and 10s Y1 T16 Y1 T17 Y1 T18	Use different objects to add equal groups. 10 x 8=80 Use Cuisenaire to aid multiplication 3 x 2 = 6 2 x 3 = 6	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there? 2+2+2=6 Use a number line and hundred square to develop understanding of multiplication as repeated addition and to record their multiplication of 2s, 5s and 10s. 	Count in multiples of a number aloud 2, 4, 6, 8, 10 5, 10, 5, 20, 25, 30 Write addition sentences to describe objects and pictures. 2+2+2=6

17
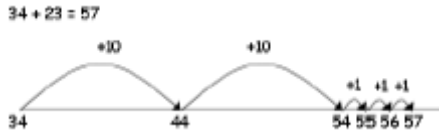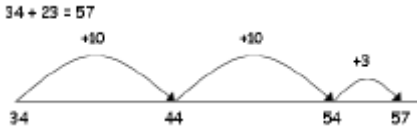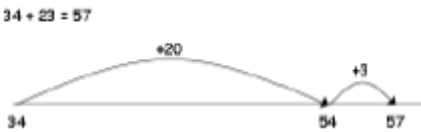
	Doubling: Y1 doubling numbers to 10 Y2 doubling numbers up to 20 + 20	Use practical activities to show how to double a number. double 4 is 8 4 × 2 = 8	Draw pictures to show how to double a number. Double 4 is 8 Double 231	Partition a number and then double each part before recombining it back together. 16 10 6 ×2 ×2 20 12
	Arrays– showing commutative multiplication Y1 T19 Y2 T21	Children begin to make connections between arrays, number patterns and counting in 2s, 5s and 10s. Create arrays using counters/cubes to show multiplication sentences.	Draw arrays in different rotations to find commutative multiplication sentences. 4×2=8 2×4=8 2×4=8 4×2=8 Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition. 5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 × 3 = 15 3 x 5 = 15

2 x 5 = 5 x 2
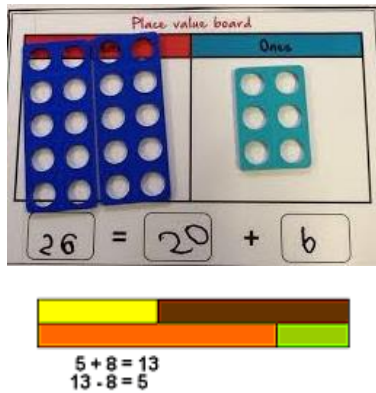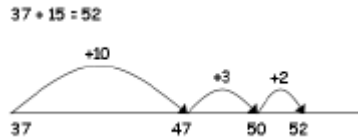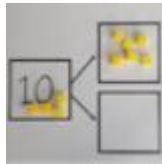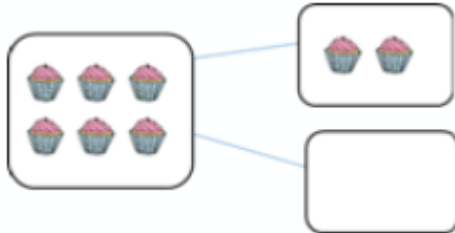
3 x 5   5 x 3?   5 x 3

Children to be able to use an array to write a range of calculations e.g.

2 x 5 = 10

5 x 2 = 10

2 + 2 + 2 + 2 + 2 = 10

5 + 5 = 10

Progression of Calculation	Key Language/ Symbols	Key Maths Resources
Doubling of numbers to 10 in Year 1 Doubling numbers up to 20 + 20 in Year 2 Counting in 2s, 5s and 10s	Multiplication Groups of Multiply Lots of Multiplied by Sets of Multiple Double Times The product of Is equal to/ is the same as	Cuisenaire Arrays Numicon Bead Strings Cubes Counters
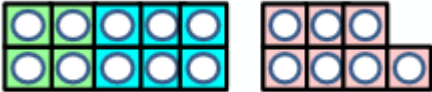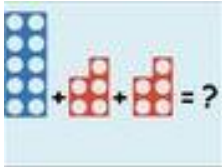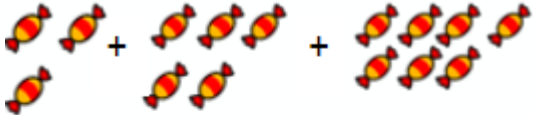
	Objective	Concrete	Pictorial	Abstract
Year 2	Partitioning Y2 T9	Children use known addition facts to derive new facts up to 100. E.g. Use knowledge that 4 + 3=7 to know 40 + 30=70 Children use knowledge of place value to partition two digit numbers. Children partition in different ways e.g. 23 = 20+3 and 23 = 10 + 13	Use partitioning knowledge to support calculation. 34 + 23 = 57 Children to become more efficient by adding the units in one jump (by using the known fact 4 + 3= 7). 34 + 23 = 57 Children move on to adding the tens in one jump and the units in one jump. 34 + 23 = 57 Children to be supported in	34 + 23 = 57
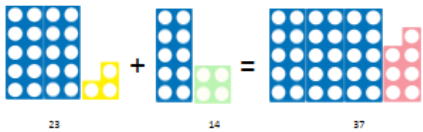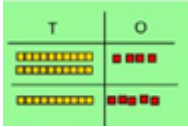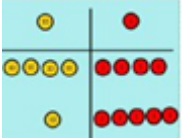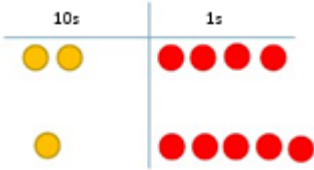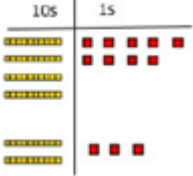
		26 = 20 + 6 5 + 8 = 13 13 - 8 = 5	understanding that bridging through ten can lead to more efficient calculation.	
	Part Part Whole model (to support finding the inverse to solve missing number problems) Y2 T14	Link to addition. Use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 − 6 =	Use pictorial representation of objects and the bar model to show the part/ part/ whole model. Whole / Part / Part Part + Part = Whole Whole − Part = Part	Move to using numbers within the part whole model
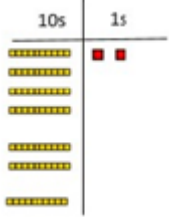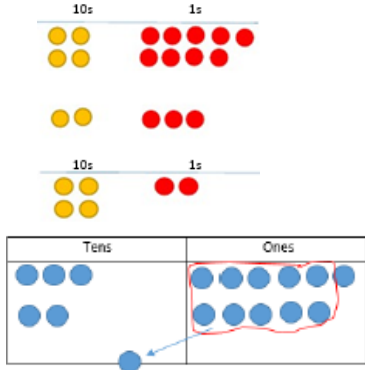
Year 2	Adding 3 single digit numbers Y2 T10d	Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit 4 + 7 + 6 = 17 Put 4 and 6 together to make 10. Add on 7. Alternative uses with Numicon.	Add together three groups of objects. Draw a picture to recombine the groups to make 10.	Combine the two numbers that make 10 and then add on the remainder.

Column method without regrouping Y2 T10c	Adding Add together the ones first, then add the tens. Use the Base 10 blocks first before moving onto place value counters. 24 + 15 = 44 + 15 =	After physically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. 	24 + 15 = 39 24 + 1 5 ――― 39
Column method with regrouping Y2 T10	Make both numbers on a place value grid. 49 + 23 = 72 	Using place value counters, children can draw the counters to help them to solve additions. 49 + 23 = 72	40 + 9 20 + 3 ――――― 60 + 12 = 72
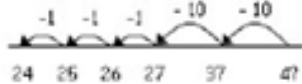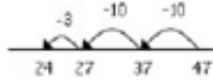
Add up the units and exchange 10 ones for 1 ten.

Looking for ways to make 10

36 + 25=

30 + 20 = 50
5 + 5 = 10
50 + 10 + 1 = 61

1   5

Formal method:

36
+25
61
 1

Progression of Calculation	Key Language/ Symbols	Key Maths Resources
TU+U (not crossing 10)	Addition	Numicon
TU+T (not crossing 10)	Equal to	Base 10/ Diennes Equipment
TU+U (bridging through 10)	Same as	Bead Strings
TU + 'teens number'	Number bonds	Number lines
TU+TU (not crossing tens)	Missing Number	Counting equipment
TU+TU (crossing tens)	One hundred more	Cuisenaire
U+U+U	Sum	Hundred Square
	Total	Place Value arrows
	Parts and wholes	
	Plus	
	Add	
	Altogether	

YEAR 2 CALCULATION GUIDANCE: SUBTRACTION

	Objective	Concrete	Pictorial	Abstract
Year 2	Counting Back Y2 T11	Continue to use a bead string 31 – 15 = Use Cuisenaire Rods 13-7 	Children use visual resources to begin to draw their own empty number lines to support calculation. Children first counting back in tens and ones. Children then become more efficient 47 - 23 = 24 by subtracting the units in one jump (by using the known fact 7 – 3 = 4). 47 - 23 = 24 Children move to subtracting the tens in one jump and the units in one jump.	Children use known facts e.g. 20 – 9 by using fact 20 – 10.

47 – 23 = 24

Children to be supported in understanding that bridging through ten can lead to more efficient calculation.

42 – 25 = 17

| Year 2 | Column method without regrouping

Y2 T11c | Use Base 10 to make the bigger number then take the smaller number away.

Show how you partition numbers to subtract. Again make the larger number first. | Draw the Base 10 (or place value counters) alongside the written calculation to help to show working.

 | 47 – 24 = 23

$$\begin{array}{r} 40+7 \\ - \;20+4 \\ \hline 20+3 \end{array}$$

This will lead to a clear written column subtraction.

 |

| Column method with regrouping

Y2 T1Ic | Use Base 10 to start with before moving on to place value counters.
45-26 =

STEP 1: Make the larger number with the place value counters

STEP 2: Start with the ones, can I take away 6 from 5 easily? I need to exchange 1 of my tens for 10 ones.

STEP 3: Subtract the ones, then the tens. | Represent the Base 10 pictorially

 | Children begin to exchange in the expanded method

Children only move onto formal column methods if place value is secure.

It's crucial that the children understand that when they have exchanged the 10 they still have 45
45 = 30 + 15 |
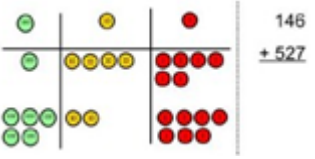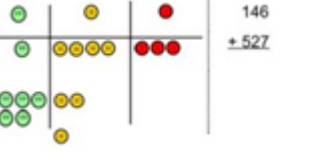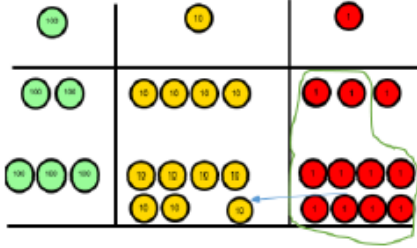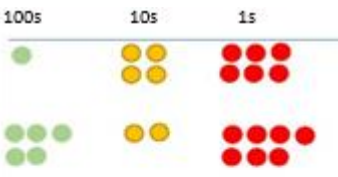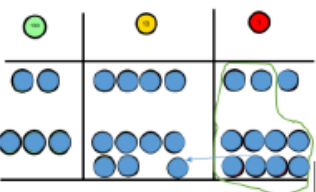
		Progression of Calculation	Key Language/ Symbols	Key Maths Resources
		TO-O (not crossing 10s) TO - O (crossing 10s) TO - T TO - TO (not crossing a ten) TO – TO (crossing a ten)	Subtract Equals Difference Number bonds Missing Number One hundred less Take away Less than Minus Fewer Decrease	Base 10/ Diennes Bead strings Counting equipment Cuisenaire Place Value Counters

YEAR 3, YEAR 4, YEAR 5 AND YEAR 6 CALCULATION GUIDANCE: ADDITION

Year	Objective	Concrete	Pictorial	Abstract
Year 3	Partitioning Y3 T4	Know place value to 3 digits. 400 50 8	Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate. Children will count on from the largest number irrespective of the order of the calculation and partition the 2nd number only. 38 + 86 = 124 +30 +4 +4 86 116 120 124 Compensation. 73 + 49 = 122 73 + 50 - 1 = 122 +50 -1 73 122 123	Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

| Year 3 and Year 4 | Column method with regrouping

Y3 T10
Y4 T14 | Make both numbers on a place value grid.
$$\begin{array}{r}146\\+527\end{array}$$
Add up the units and exchange 10 ones for 1 ten.
$$\begin{array}{r}146\\+527\end{array}$$

As children move on to money, decimal place value counters as well as Base 10 can be used to support learning.
£1.05 + £0.78 | Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.

If the children are completing a word problem, draw a bar model to represent what it's asking them to do. | Expanded Method
$$\begin{array}{r}100 + 40 + 6\\500 + 20 + 7\\\hline 600 + 70 + 3 = 673\end{array}$$
As the children progress, they will move from the expanded to the compacted method.
$$\begin{array}{r}146\\+\ 527\\\hline 673\\1\end{array}$$
$$\begin{array}{r}243\\+368\\\hline 611\\1\ \ 1\end{array}$$
As the children move on, introduce decimals involving money.
$$\begin{array}{r}£\ .\ p\\1.05\\+\ 0.78\\\hline 1.83\\1\end{array}$$ |
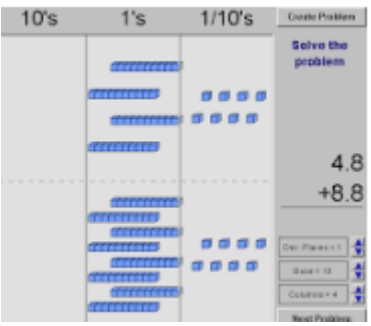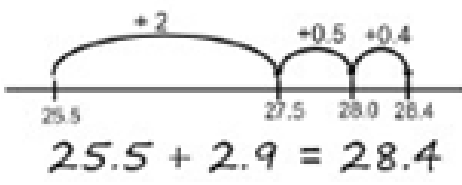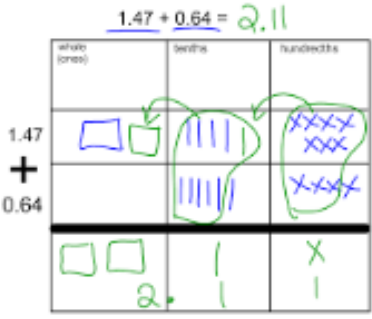
NB: By year 4 children will progress onto adding 4 digit numbers.

?	
243	358

Addition of money needs to have £ and p added separately

70p   5p   3p

£1.05   £1.75   £1.80   £1.83
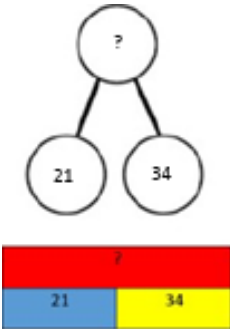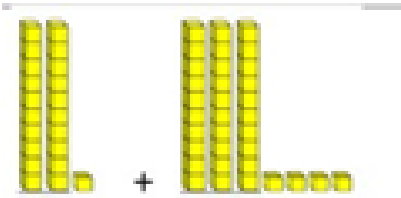
| Year 5 and Year 6 | Column method with regrouping (including decimals) Y5 T10 Y5 T49 Y6 T12 | Consolidate understanding using numbers with more than 4 digits and extend by adding numbers with up to 3 decimal places. | Children will use the empty number line method to support adding decimals. $25.5 + 2.9 = 28.4$ | Compact method of addition (FORMAL METHOD) Method to be extended to numbers with at least four digits and decimals (including missing place holders). Children may include zero place-holders to aid understanding of place value. Children to go back to the expanded method as necessary, in particular when going up to 1000 and 2-decimal places. Resources to be used to support understanding of the compact method where necessary e.g. straws, Diennes, place value counters etc. (See Year 4). |
| | | Progression of Calculation Year 3 Review from Y2: TU+TU (with and | Key Maths Resources Base 10/ Diennes Equipment Place Value Counters | Key Language/ Symbols Addition Equal to |

| | | without crossing ten) TO + TO (answer is greater than 100) HTO + T HTO + TO HTO + HTO <u>Year 4</u> HTO + HTO (Crossing 100) ThHTO + ThHTO £O.t h + £O. t h £TO.t h + £TO. t h <u>Progression of Calculation</u> <u>Year 5</u> Addition of numbers beyond 1000s Addition of numbers with up to two decimal places. Addition of numbers with at least 4 digits and decimals with various missing place- holders <u>Year 6</u> Addition of numbers with any number of digits. Addition of two or more numbers with at least 4 digits and 3 decimal places. Addition of two or more numbers with at least 4 digits of various sizes and | Place Value arrows Empty Number Line | Same as Number bonds Missing Number One hundred more Hundreds Boundary Pounds and Pence Inverse Ones Boundary Tenths Boundary Sum Total Parts and wholes Plus Altogether |
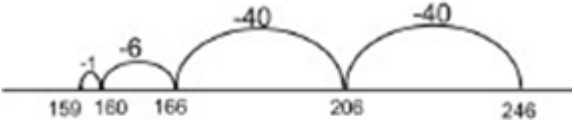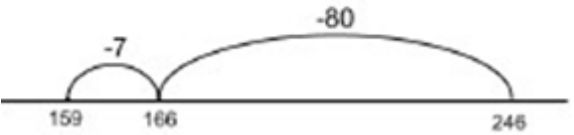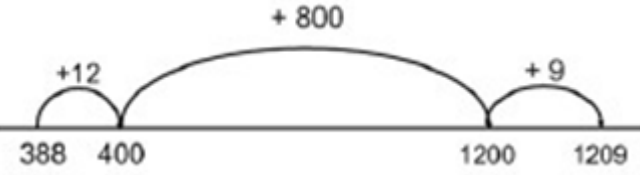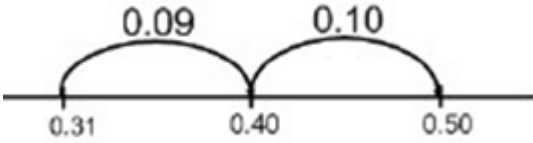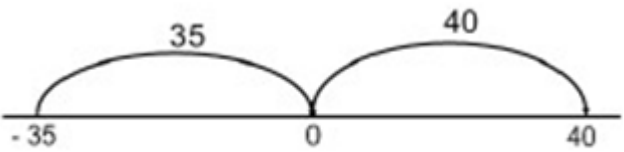
	varied decimal places. (E.g. 401.2 + 26.85 + 13 =)		

<table>
<tr><td colspan="4" align="center">Fluency variation, different ways to ask children to solve 21+34:</td></tr>
<tr>
<td></td>
<td>Sam saved £21 one week and £34 another. How much did he save in total?

21+34=55. Prove it! (reasoning but the children need to be fluent in representing this)</td>
<td>21
+34
―――

21 + 34 =

☐ = 21 + 34

What's the sum of twenty one and thirty four?</td>
<td>
Always use missing digit problems too:

<table>
<tr><td>Tens</td><td>Ones</td></tr>
<tr><td>◉ ◉</td><td>●</td></tr>
<tr><td>◉ ◉ ◉</td><td>?</td></tr>
<tr><td>?</td><td>4</td></tr>
</table>
</td>
</tr>
</table>

	Objective	Concrete	Pictorial	Abstract
Year 3 to Year 6	Counting back Y3 TII Y5 TII		**Year 3** Children will continue to use empty number lines with increasingly large numbers. E.g. 246-87= or with confidence **Year 5** Complementary Addition E.g. 1209 – 388 = Empty Number Lines and Decimals Children will apply this method to subtract	Children will be encouraged to use informal pencil and paper methods (jottings).

decimals including bridging through tenths and ones.

E.g. 0.5-0.31 = 0.19

Year 6

Empty Number Line for Negative Numbers

An empty number line will be used to show differences between negative numbers and positive and negative numbers.

E.g. What is the difference between 40 and -35?

| Y6 T5 Y6 T11 | | | | |

| Year 3 to Year 6 | Column method with regrouping | Use Base 10 to start with before moving on to place Value counters. Start with one exchange before moving onto subtractions with 2 exchanges. | Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make. When confident children can find their own way | Children can start their formal written method by partitioning the number into clear place value columns. |

Y3 TII
Y4 TI5
Y5 TII

Make the larger number with the place value counters.

Calculations
234
- 88
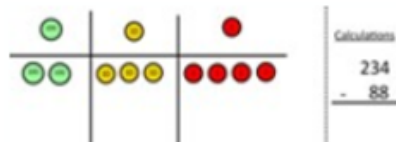
Start with the ones, can I take 8 away from 4 easily? I need to exchange one of my tens for ten ones.

Calculations
234
- 88
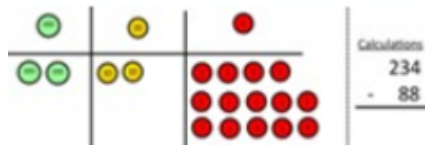
Now I can subtract my ones.

Calculations
234
- 88

836 - 254 = 582
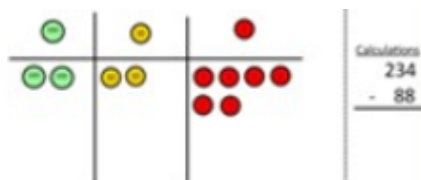
Moving forward the children use a more compact method.

728 - 582 = 146
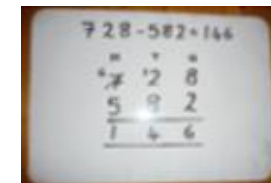
Year 5
Expanded Method to FORMAL compact method
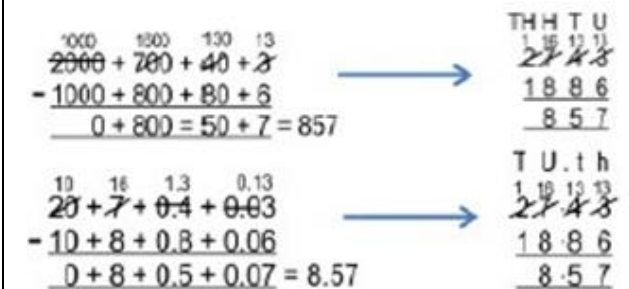
This will lead to an understanding of subtracting any number including decimals and money.

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.

Calculations
234
- 88
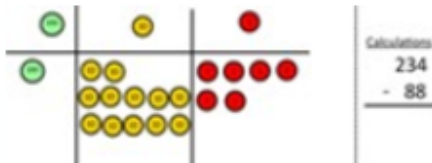
Now I can take away my eight tens and complete my subtraction.

Calculations
'234
- 88
146
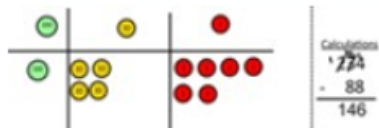
Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

$$
\begin{array}{r}
5121 \\
2\ \ 6\!\!\!/\ \ 3\!\!\!/\ .\ \ 0 \\
-\quad 2\ \ 6\ .\ 5 \\
\hline
2\ \ 3\ \ 6\ .\ 5 \\
\end{array}
$$

£8.95 =   8 + 0.9 + 0.05   leading to
-£4.38   - 4 + 0.3 + 0.08

     =   8 + 0.8 + 0.15   (adjust from T to U)   8.85
        - 4 + 0.3 + 0.08   -4.38
        ────────────────
          4 + 0.5 + 0.07

        = £4.57

		Progression of Calculation	Progression of Calculation	Key Language/ Symbols
		Year 3	Year 5	Inverse
		Empty Number Line	TH H T U – TH H T U	Exchange
		HTO-TO (crossing100s)	U.t – U.t	Subtract
		HTO-HTO	U.t h – U. t h	Equals
		Expanded Column Method	U. t h – U.t	Difference
		HTO-HTO	U. t – U .t h	Number bonds
		Stage 1 : no exchange	Larger numbers involving decimals	Missing Number
		Stage 2: Exchange T to O	Year 6	One hundred less
		Stage 3: Exchange H to T	Refine Year 5	Take away
		Year 4	Difference between two negative integers.	Less than
		HTO – TO; HTO-HTO	Difference between positive and negative integers.	Minus
		Stage 4 : Exchange of T to O and H to T		Fewer
		Stage 5 £0.t t - £0.t h and Handling of zero place holders		Decrease
				Key Maths Resources
				Empty Number Line
				Diennes/Base 10/Big Base
				Calculation Mat
				Place Value Counters

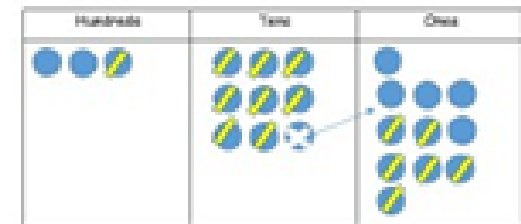| Fluency variation, different ways to ask children to solve 391-186:			
Raj spent £391, Timmy spent £186. How much more did Raj spend? I had 391 metres to run. After 186 I stopped. How many metres do I have left to run?		391 – 186 ☐ = 391 – 186 391 -186 ――― Find the difference ebtween 391 and 186 Subtract 186 from 391. What is 186 less than 391?	What's the calculation? What's the answer?
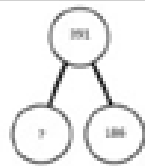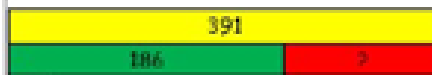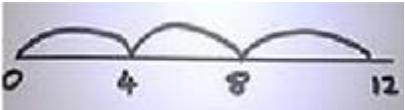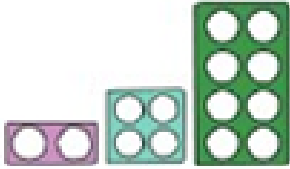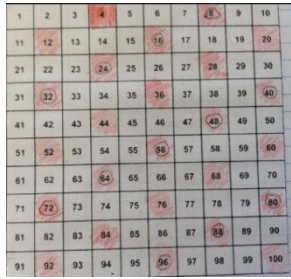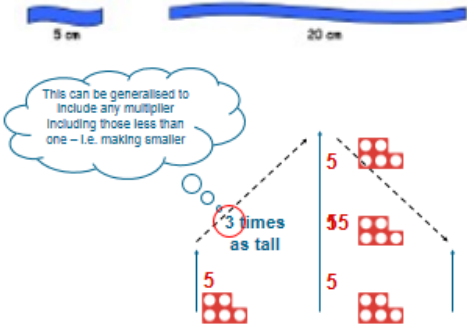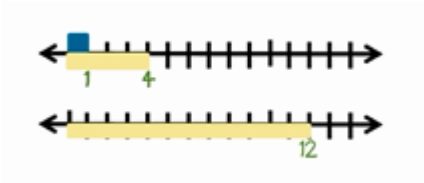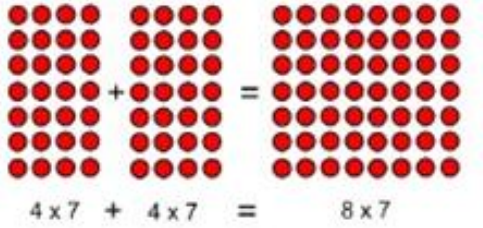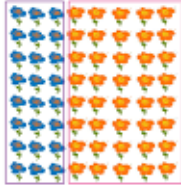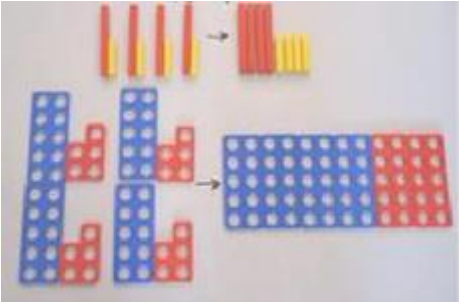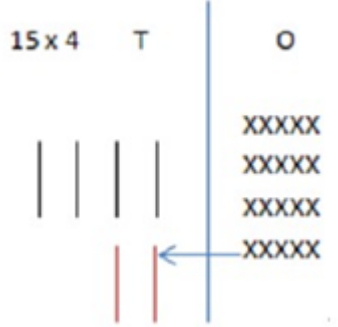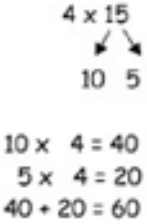
	Objective	Concrete	Pictorial	Abstract
Year 3	Repeated addition: Counting in 3s, 4s and 8s and continue to consolidate 2s, 5s and 10s Y3 T16	Repeated grouping/repeated addition (does not have to be restricted to cubes) 3 x 4 or 3 lots of 4 Use number lines to show repeated groups- 3 x 4 Children should continue to use bead bars to show repeated addition	Children to represent the practical resources in a picture e.g. XX XX XX XX XX XX Use of a bar model for a more structured method Represent this pictorially alongside a number line E.g.: Abstract number line 3 x 4 = 12	3 x 4 4 + 4 + 4 3 x 4 = 12

	Doubling To see links between 2, 4 and 8 multiplication facts			2 x 4 is double 2 x 2 2 x 2 is double 1 x 2
	Scaling Y3 T25	Children must be given opportunities to answer problem-solving questions related to the 'Scaling' model of multiplication. E.g. Find a ribbon that is 4 times as long as the blue ribbon. 	4 scales up 3 times to make 12 can be shown on a number line. 	3 x 4 = 12
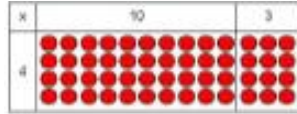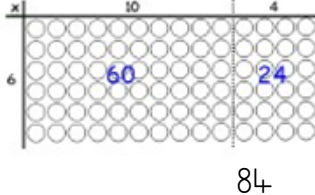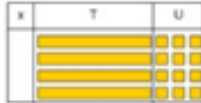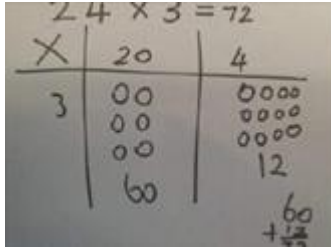
	Arrays to understand the associative laws of multiplication	Arrays can also be used to support the associative law of multiplication. E.g. $4 \times 7 + 4 \times 7 = 8 \times 7$	Reason pictorially. Task Mary decided to plant her garden as shown in the picture below. Which student correctly described the garden with an equivalent expression? Explain your answer. Student A said: 8×8 Student B said: $(8 \times 3) + (8 \times 5)$ Student C said: $4 \times 4 + 8 \times 8$ Student D said: $(3 \times 6) + (5 \times 2)$	$(3 \times 8) + (5 \times 8) = 64$
	Partitioning Y3 T23	Partition to multiply – Numicon, Cuisenaire rods and Base 10 resources 4×15	Children to represent the concrete manipulatives in a picture e.g. base 10 can be represented like: A number line can also be used	Children to be encouraged to show the steps they have taken 4×15 $10 \quad 5$ $10 \times \ 4 = 40$ $5 \times \ 4 = 20$ $40 + 20 = 60$ $38 \times 5 = (30 \times 5) + (8 \times 5)$ $= 150 + 40$ $= 190$
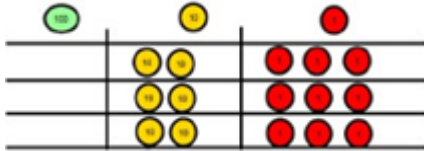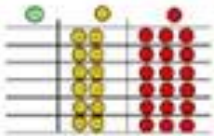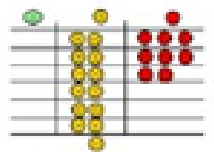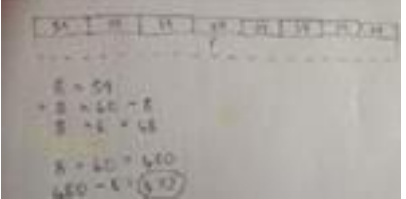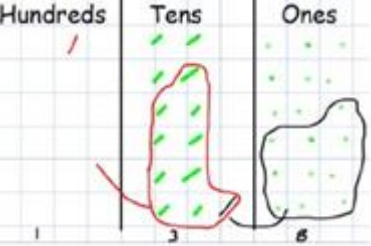
| Year 4 | Y4 Arrays to introduce the Distributive Law

Y4 T26 | The Distributive Law says that multiplying a number by a group of numbers added together is the same as doing each multiplication separately

$3 \times (3 + 3)$ = 3×3
        + 3×3

 | Example: $3 \times (2 + 4) = 3 \times 2 + 3 \times 4$

So the "3" can be "distributed" across the "2+4" into 3 times 2 and 3 times 4.

$3 \times (2+4)$ $3 \times 2 + 3 \times 4$ | $3 \times (2 + 4) = 3 \times 2 + 3 \times 4$ |

| Grid method
U x TU(grid)
U x HTU
(grid)

Y4 T23
Y4 T24 | Show the link with arrays to first introduce the grid method. 13 x 4
4 rows of 10
4 rows of 3
(4 x 10) + (4 x 3)
40 + 12
52

 (6 x 10) +
(6 x 4)
60 +
24
84

Move on to using Base 10 to move towards a more compact method

 4 rows of 13 | Children can represent the work they have done with place value counters in a way that they understand pictorially.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.

 | Children to record what it is they are doing to show understanding

613 x 5 =
We partition 613 into 600 and 10 and 3 and put it in a table.

| x | 600 | 10 | 3 |
| 5 | 3000 | 50 | 15 |

Add up 3000, 50 and 15 to make 3065.
613 x 5 = 3065 |

Formal Column Method U x TU (formal) U x HTU (formal) Y4 T23 Y4 T24	FORMAL COLUMN METHOD with place value counters or Base 10 to show how we are finding groups of a number. We are multiplying by 3 so we need 3 rows (at the first stage-no exchanging) 3 x 23 Make 23, 3 times. See how many ones, then how many tens.	Children represent counters in a pictorial way	Children to record what it is they are doing to show understanding

FORMAL COLUMN METHOD with place value counters or Base 10 to show how we are finding groups of a number. We are multiplying by 3 so we need 3 rows (at the first stage-no exchanging) 3 x 23

Make 23, 3 times. See how many ones, then how many tens.

Move on to Place Value Counters or Base 10 making an exchange. E.g. 6 x 23

Step 1: get 6 lots of 23

Step 2: 6 x 3 is 18. Can I make an exchange? Yes! Ten ones for one

Children represent counters in a pictorial way

Tens | Ones
6 | 9

Bar modelling can support solving problems with multiplication alongside the formal written methods.

Hundreds | Tens | Ones

Children to record what it is they are doing to show understanding

3 x 23   3 x 20 = 60
  / \   3 x 3 = 9
20   3   60 + 9 = 69

    23
  X 3
    69

The aim is to get to the formal method but the children need to understand how it works.

$6 \times 23 =$

    23
  × 6
   138
   1 1

ten....

Step 3: 6 x 2 tens and my extra ten is 13 tens. Can I make an exchange? Yes! Ten tens for one hundred...
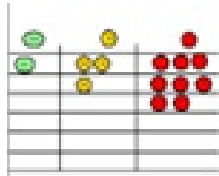
Step 4- what do I have I each column?

Move on to HTU with place value counters/ Base 10 We are multiplying by 4 so we need 4 rows.
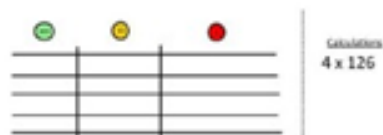
Calculations
4 x 126

Fill each row with 126.
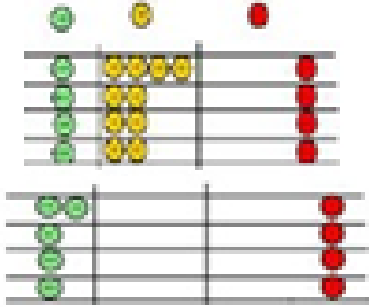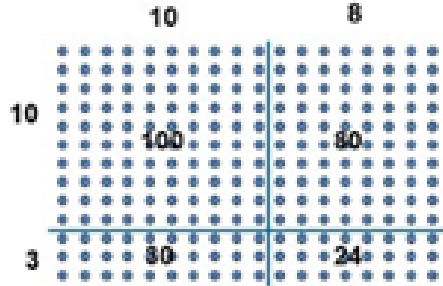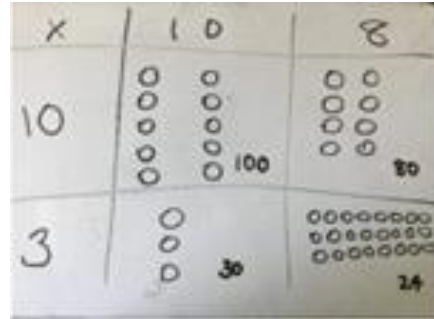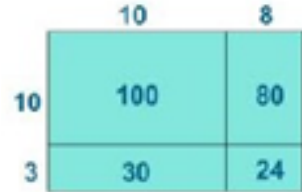
Calculations
4 x 126

342×7 becomes

$$
\begin{array}{cccc}
 & 3 & 4 & 2 \\
\times & & & 7 \\
\hline
2 & 3 & 9 & 4 \\
 & 2 & 1 &
\end{array}
$$
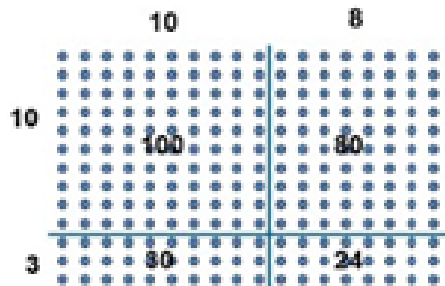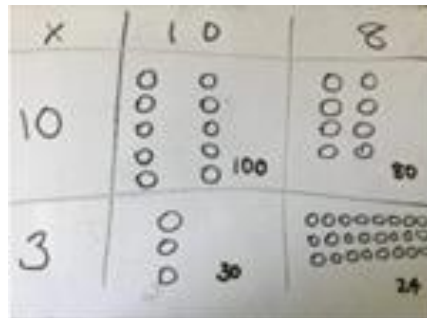
Answer: 2394

		Add up each column, starting with the ones making any exchanges needed.		
		4 × 126 = 504		
Year 5	Grid method TU x TU U x U.t Y5 T25	Grid method will continue to support understanding of short and long multiplication.		56 x 27 Est. 60 x 30 = 1800

				Move on to decimals 3 x 4.9 Est. 3 x 5 = 15 | x	4	0.9
3	12	(2.7)	 $\begin{array}{r}12.0\\+\ 2.7\\\hline 14.7\end{array}$ It may help children think of this as *3 x 9 tenths = 27 tenths*, then use their understanding of place value to help form 2.7.
	Expanded method	Show the link with arrays to first introduce the expanded method. 10 8 10 100 80 3 30 24		Children will transfer their understanding to the expanded column method – starting with least significant digit first $\begin{array}{r}56\\\times\ 27\\\hline 42\\350\\120\\+\ 1000\\\hline 1512\end{array}$ (7 x 6) (7 x 50) (20 x 6) (20 x 50) Children complete units part of the calculation first. Steps in brackets may support understanding of the method If children are confident in this method they may begin to use Long Multiplication (see Year 6) reverting to the grid method as necessary to support

				understanding of more difficult calculations.
	Y5 & Y6 Formal Method (Long Multiplication) Y5 T24 Y6 T7			Children may continue to use the grid method as necessary to maintain understanding of place value when multiplying larger numbers and understanding the process of finding partial products. Move understanding to column method as soon as secure in their understanding of the expanded method (see Year 5). If necessary use expanded method as a stepping stone to support understanding.
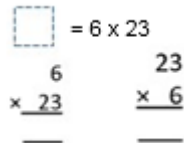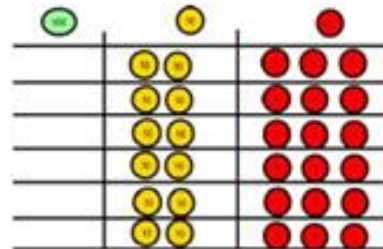
				1 2 4
				× 2 6
				7 4 4
				₁ ₂
				2 4 8 0
				3 2 2 4
				1 1
				Answer: 3224
				When children start to multiply 3d x 3d and 4d x 2d etc, they should be confident with the abstract: What Year group is it Year 7: This moves to the more compact method.
		Progression of Calculation Year 3 O x (2.3.4.5.8.10) (1-20) x (2,3,4,5,8,10) – beyond 20 when confident. Multiply 1 or 2 digit numbers by 10 and 100. Year 4 O x TO(grid) O x HTO (grid) Multiplication of numbers by 10,100 and 1000.	Key Language/ Symbols Multiplication Groups of Multiply Lots of Multiplied by Sets of Multiple Factor Product Double Times	Key Maths Resources Base 10 Arrays Numicon Bead Strings Cubes Counters Multiplication Squares Empty Number Line Place Value Counters Cuisenaire rods
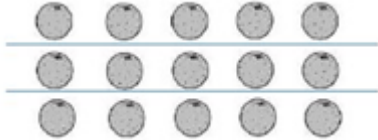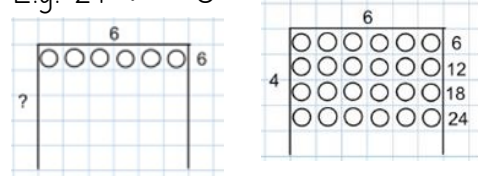
		Multiplication of 3 numbers to (TO) Knowledge of all multiplication facts up to 12x12. <u>Year 5</u> TO x TO O x O.t TH H T O x O (Long MOltiplication) TH H T O x TO (Long MOltiplication) <u>Year 6</u> H TOxTO O x O. t h	The product of Is equal to/ is the same as	

Fluency Variation, different ways to ask children to solve 6 x 23
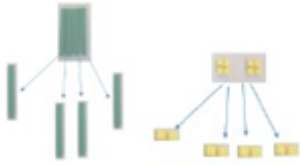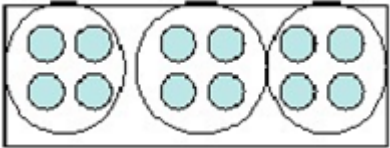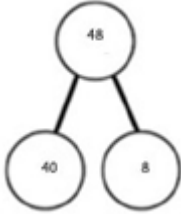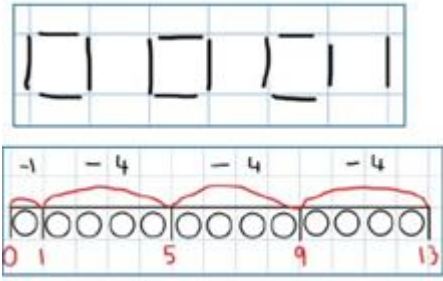
Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week? Tom saved 23p three days a week. How much did he save in 2 weeks?	23 23 23 23 23 23 ? With the counters, prove that 6 X 23 = 138 Why is 6 x 23 = 32 x 6?	Find the product of 6 and 23 6 x 23 = □ = 6 x 23 6 23 × 23 × 6	What's the calculation? What's the answer?
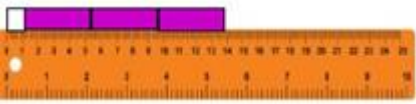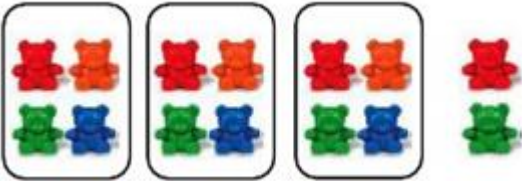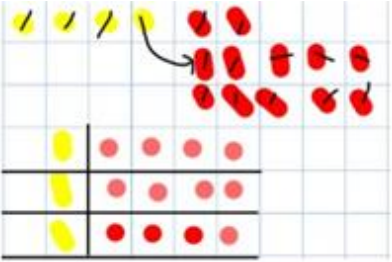
YEAR 3, YEAR 4, YEAR 5 AND YEAR 6 CALCULATION GUIDANCE: DIVISION

	Objective	Concrete	Pictorial	Abstract
Year 3	Division with arrays Y3 T20 Y3 T21 Y3 T22	Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences. 	Find the inverse of multiplication and division sentences by creating four linking number sentences. 5 x 3 = 15 3 x 5 = 15 15 ÷ 5 = 3 15 ÷ 3 = 5 Arrays and known facts can be used to support children in answering missing number calculations. E.g. 24 ÷ = 6

53

TO ÷ O with no remainders Y3 T23	Using base 10 and SHARING 48 ÷ 4 = 12 Start with the tens	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups. Encourage them to move towards counting in multiples to divide more efficiently. 12 ÷ 3 = 4 	48 ÷ 4 4 tens ÷ 4 = 1 ten 8 ones ÷ 4 = 2 ones 10 + 2 = 12
TO ÷ O with remainders	13 ÷ 4 = 3 remainder 1 Use of lollipop sticks to form wholes Use of Cuisenaire rods and rulers (using repeated subtraction)	Children to have chance to represent the resources they use in a pictorial way e.g. see below 13 ÷ 4 = 3 r 1 Children use repeated subtraction on a number line then see how many more	13 ÷ 4 = 3 remainder 1 Children to count up their times tables facts in their heads

		$14 \div 3 = 4 \text{ r } 2$ $10 \div 3 = 3r1$	you need to jump to find a remainder. Draw dots and group them to divide an amount and clearly show a remainder. $14 \div 4 = 3 \text{ r } 2$ 	
Year 3 to Year 6	Short Division Y3 TO ÷ O Y4 TO ÷ O Y5 HTO ÷ O/ TH H T O ÷ O Y6 (with decimal and fraction remainders) TH H T O ÷ O	Year 3: TO ÷ O Use place value counters to divide using the short division method alongside. $42 \div 3 = 14$ Make 42.	$42 \div 3 = 14$ 	Begin with divisions that divide equally with no remainder. $42 \div 3$ $42 = 30 + 12$ $30 \div 3 = 10$ $12 \div 3 = 4$ $10 + 4 = 14$

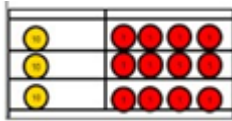|				
	Y3 T23 Y4 T19 Y5 T25 Y6 T8/T9	Share the 4 tens between 3. Can we make an exchange with the extra 10? Exchange the ten for 10 ones and share out 12 ones We look at how many are in each group which is 14. Y4 HTO ÷ O Use of the 'bus stop method' using grouping and counters. Key language for grouping- how many groups of X can we make with X hundreds'- this can also be done using sharing! 615 ÷ 5	The 'bus stop' can easily be represented pictorially, till the children no longer need to do it. It can also be done to decimal places if you have a remainder!	When ready progress to short formal method in Year 3 TO ÷ O $3 \overline{\smash{)}7\,^{1}5}$ = 2 5 Progress to short formal method with remainders in Year 4 $3 \overline{\smash{)}7\,^{1}7}$ = 2 5 r2 Progress to HTO ÷ O (Y4) $5 \overline{\smash{)}6\,^{1}1\,^{1}5}$ = 123 Progress to Th HTO ÷ O (Y5) $5 \overline{\smash{)}6\,^{1}7\,^{2}4\,^{4}5}$ = 1 3 4 9
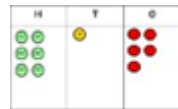
Step 1: Make 615

Step 2: Circle your groups of 5

Step 3: Exchange 1H for
10T and circle groups of 5

Step 4: Exchange 1T for

$364 \div 3 =$
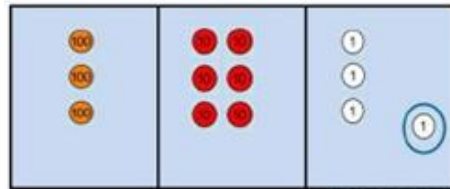
10 ones and circles groups of 5

57

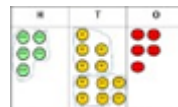| Year 6	Long Division			
	Y6 T8	2544 ÷ 12 How many groups of 12 thousands do we have? None Exchange 2 thousand for 20 hundreds. How many groups of 12 are in 25 hundreds? 2 groups. Circle them. We have grouped 24 hundreds so can take them off and we are left with one. Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2. Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2	Children to represent the counters, pictorially and record the subtractions beneath.	Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds. Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left. Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many tens I have left. Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.

		Progression of Calculation Year 3 TO ÷ (2,3,4,5,8,10) TO ÷ (2,3,4,5,8,10) Year 4	Year 5 HTU÷U=TU HTU÷U=HTU THTU÷ U = Year 6	Key Language/ Symbols Half Dividing/ Grouping/ Sharing Divided by/ into Equal groups of

		TO ÷ O = including remainders.	THTU ÷U (with decimal and fraction remainders, depending on context) HTU ÷ TU THTU ÷TU U.t ÷U TU.t ÷U	Inverse Is equal to/ is the same as Key Maths Resources Cubes Base 10 Place Value Counters Arrays Numicon Number Lines Cuisenaire Rods
		Fluency variation, different ways to ask children to solve 615 ÷ 5:		
Using the part whole model below, how can you divide 615 by 5 without using the 'bus stop' method? 	I have £615 and share it equally between 5 bank accounts. How much will be in each account? 615 pupils need to be put into 5 groups. How many will be in each group?	$615 ÷ 5 =$ ☐ $= 615 ÷ 5$ How many 5's go into 615?		What's the calculation? What's the answer?